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Optimal Load Shedding in Electric Power Grids

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Preprint ANL/MCS-P5408-0915

September, 2015

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research, under Contract DE-AC02-06CH11357.

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***Abstract*— We consider the optimal load-shedding problem in electric power systems where a number of transmission lines are to be taken out of service. The nonlinear power flow equations and the binary decision variables lead to a mixed-integer nonlinear program. We show that the load-shedding problem has separable structure that can be exploited by using the alternating direction method of multipliers. We show that the subproblems in the alternating method can be solved efficiently. Numerical experiments with the IEEE 118-bus test case illustrate the effectiveness of the developed approach. Our computational results suggest that removing transmission lines between load buses results in less load shedding.**

Keywords: Alternating direction method of multipliers, load-shedding problem, mixed-integer nonlinear programs, power systems, separable structure.

I. Introduction

Redundancy of interconnection in power systems is known to help prevent cascade blackout [1]. On the other hand, a new study suggests that having too much interconnectivity in power networks can result in excessive capacity, which in turn fuels larger blackout [2]. Therefore, a balance between the operational robustness and the network interconnectivity is important for power grid operations.

Traditionally, contingency analysis in power grids has focused on the severity of line outages using linearized power flow models; see [3]. Recent years have seen vulnerability analysis of line outages using nonlinear power flow models; see [4]–[6]. The objective of these studies is to identify transmission lines whose removal leads to the maximum damage (e.g., in load shedding) to power systems. The optimal transmission switching is a related line of research that focuses on the switching of transmission lines to reduce congestion in power grids [7], [8].

While identifying vulnerability in power systems is important, identifying redundancy in power systems is equally important. By redundancy, we mean the components (e.g., transmission lines) in the power grid whose removal does not change the grid operation significantly. Suppose that the power grid is operating at its nominal point with a balance between load and power generation. Consider the situation in which the grid operator must temporarily remove a number of transmission lines because of, for example, maintenance or security examination [9]. The change of network topology implies a change of the operating point. Following [4]–[6], we measure the severity by the amount of load that must be shed. Our objective is to identify a prespecified number of lines whose removal results in minimum load shedding subject to the nonlinear power flow constraints.

To this end, we formulate the optimal load-shedding problem, which contains binary decision variables and nonlinear AC power flow constraints. Therefore, it falls into the class of mixed-integer nonlinear programs (MINLPs). This challenging optimization problem is beyond the capacities of the state-of-the-art MINLP solvers even for small power systems. We show that the problem has a separability structure; that is, all decision variables are separable except for the coupling nonlinear power flow constraints. To exploit this structure, we develop an approach based on the alternating direction method of multipliers (ADMM). While this approach does not guarantee to converge to an optimal solution of the nonconvex problem, our numerical experiments on the IEEE 118-bus test case show promising results.

Our presentation is organized as follows. In Section II, we introduce the notation and formulate the optimal load-shedding problem. In Section III, we study the separable structure of the load-shedding problem. In Section IV, we describe the ADMM algorithm and show that each optimization subproblem can be solved efficiently. In Section V, we provide numerical results for the IEEE 118-bus test case. In Section VI, we conclude the paper by summarizing our contributions.

II. Optimal Load-Shedding Problem

We consider a lossless power network with n buses and m lines. A line l connecting bus i and buse j can be described by a vector $e_l \in \mathbb{R}^n$ with 1 and -1 at the ith and jth elements, respectively, and 0 everywhere else. Let $E = [\, e_1 \cdots e_m \,] \in \mathbb{R}^{n \times m}$ be the incidence matrix that describes m transmission lines of the network, and let $D \in \mathbb{R}^{m \times m}$ be a diagonal matrix with the lth diagonal element being the admittance of line l. For a lossless power grid with fixed voltage at the buses, the active power flow equation can be written in matrix

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This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research, Applied Mathematics program under contract number DE-AC02-06CH11357.

form [4]

$$ED\sin(E^T\theta) \;=\; P,$$

where $\theta \in \mathbb{R}^n$ is the phase angles and $P \in \mathbb{R}^n$ is the real power injection at the buses. We numerate the buses such that the power injection P can be partitioned into a load vector $P_l \le 0$ and a generation vector $P_g > 0$, thus, $P = [P_l^T \; P_g^T]^T$. The sequence of buses indexed in P is the same with that of the columns of the incidence matrix E. We assume that the power system is lossless, so the sum of load is equal to the sum of generation

$$\mathbf{1}^T P \;=\; 0,$$

where $\mathbf{1}$ is the vector of all ones.

Our objective is to identify a small number of lines whose removal leads to the minimum change of total load in the power system. Let $\gamma \in \{0,1\}^m$ denote whether a line is in service or not: $\gamma_l = 1$ if line l is in service and $\gamma_l = 0$ if line l is out of service. Let $z = [z_l^T \; z_g^T]^T \in \mathbb{R}^n$, where $z_l \ge 0$ and $z_g \le 0$ are the load-shedding vector and the generation reduction vector, respectively. Then, it follows that

$$P_l \;\le\; P_l \,+\, z_l \;\le\; 0, \quad 0 \;\le\; P_g \,+\, z_g \;\le\; P_g,$$

where the upper (resp. lower) bound 0 enforces $P_l + z_l$ (resp. $P_g + z_g$) to be a load (resp. generator) vector.

Since the load shed must be equal to the generation reduction, we have

$$\mathbf{1}^T z \;=\; 0.$$

Now the active power flow equation with possible line removal can be written as

$$ED\Gamma\sin(E^T\theta) \;=\; P \,+\, z,$$

where $\Gamma = \text{diag}(\gamma)$ is a diagonal matrix with its main diagonal equal to γ.

Our objective is to identify a small number of lines in the power network whose removal will result in the minimum load shedding. Thus, we consider the following *optimal load-shedding* problem:

$$\begin{aligned}
\underset{\theta,\, z,\, \gamma,\, \Gamma}{\text{minimize}} \quad & \text{LoadShedding} \;=\; \mathbf{1}^T z_l && \text{(1a)}\\
\text{subject to} \quad & ED\Gamma\sin(E^T\theta) \;=\; P \,+\, z && \text{(1b)}\\
& \Gamma \;=\; \text{diag}(\gamma) && \text{(1c)}\\
& \gamma \in \{0,1\}^m, \quad m - \mathbf{1}^T\gamma \;=\; K && \text{(1d)}\\
& \mathbf{1}^T z \;=\; 0, \quad z \;=\; [\,z_l^T \; z_g^T\,]^T && \text{(1e)}\\
& P_l \le P_l + z_l \le 0,\; 0 \le P_g + z_g \le P_g && \text{(1f)}\\
& -\frac{\pi}{2} \;\le\; E^T\theta \;\le\; \frac{\pi}{2}. && \text{(1g)}
\end{aligned}$$

The decision variables are the phase angle θ, the reduction of load and generation z, the out-of-service line indicator γ, and the diagonal matrix Γ. The problem data are the incidence matrix E, the admittance matrix D, the real power injection at the buses P, and the number of out-of-service lines K. The angle difference between buses $E^T\theta$ takes values between $-\pi/2$ and $\pi/2$.

Note that the adaption of our model (1) to the maximum load-shedding problem is immediate. The load-shedding problem is based on the model originally introduced in [4]. Additional discussions of related models can be found in [4]–[6].

III. The Separable Structure

The load-shedding problem (1) is a nonlinear program with binary variables. One source of nonlinearity is the sinusoidal function $\sin(E^T\theta)$, and another source is the multiplication between Γ and $\sin(E^T\theta)$. Therefore, it falls into the class of mixed-integer nonlinear programs (MINLPs), which are challenging problems. In particular, finding a feasible point for MINLPs can be computationally expensive or even NP-hard [10]–[12].

The minimum load-shedding problem (1) turns out to have a separable structure that can be exploited. In what follows, we discuss this structure and develop an algorithm based on the alternating direction method of multipliers.

A closer look of (1) reveals that the only constraint that couples all variables, θ, z, γ, and Γ, is the power flow equation (1b). On the other hand, the diagonal matrix Γ is determined by γ in (1c). The binary variable γ is subject only to the cardinality constraint (1d). The load- and generation-shedding variable z is subject to the losslessness constraint (1e) and the elementwise box constraint (1f). The phase angle θ is subject only to the linear inequality constraint (1g). Therefore, the constraints in the load-shedding problem (1) are separable with respect to θ, z, γ, and Γ, if the power flow equation (1b) is relaxed.

The idea is then to dualize (1b) by introducing the Lagrangian multiplier. Let us denote the coupling constraint as

$$c(\theta, z, \gamma) \;=\; ED\Gamma\sin(E^T\theta) \,-\, (P \,+\, z).$$

Consider the minimization of the partial augmented

Lagrangian function of (1):

$$
\begin{array}{ll}
\underset{\theta,\,z,\,\gamma,\,\Gamma}{\text{minimize}} & \mathcal{L}_\rho(\theta,z,\gamma,\Gamma,\lambda) \;=\; \mathbf{1}^T z_l \\
& \quad +\, \lambda^T c(\theta,z,\gamma) \,+\, \dfrac{\rho}{2}\,\|c(\theta,z,\gamma)\|_2^2 \\
\text{subject to} & \Gamma \;=\; \text{diag}(\gamma) \\
& \gamma \,\in\, \{0,1\}^m, \quad m - \mathbf{1}^T\gamma \;=\; K \\
& \mathbf{1}^T z \;=\; 0, \quad z \;=\; [\, z_l^T \; z_g^T \,]^T \\
& P_l \,\leq\, P_l \,+\, z_l \,\leq\, 0,\; 0 \,\leq\, P_g \,+\, z_g \,\leq\, P_g \\
& -\dfrac{\pi}{2} \,\leq\, E^T\theta \,\leq\, \dfrac{\pi}{2},
\end{array}
\tag{2}
$$

where $\lambda \in \mathbb{R}^n$ is the Lagrange multiplier and ρ is a positive scalar.

Clearly, (2) is a relaxation of the load-shedding problem (1), since the power flow equation

$$
c(\theta,z,\gamma) \;=\; 0
$$

is no longer enforced in (2). The penalty of the constraint violation is controlled by the positive scalar ρ in the objective function $\mathcal{L}_\rho$. By solving the relaxed problem (2) with an appropriate choice of λ and a sufficiently large ρ, the minimizer of (2) provides a feasible solution for (1) when the residual $\|c(\theta,z,\gamma)\|_2$ is sufficiently small.

IV. ADMM Algorithm

The ADMM algorithm has proved effective for many problems; see the survey paper [13]. Among other applications in control, ADMM has been successfully applied to the design of sparse feedback gain [14], the leader selection problem in consensus networks [15], and the completion of state covariances [16].

We next develop an alternating direction method that exploits the separable structure of (2). Roughly speaking, we minimize $\mathcal{L}_\rho$ with respect to θ, z, or γ, one at a time, while fixing the other variables constant. For ease of presentation, we introduce the following indicator functions:

$$
\phi_1(\gamma) \;=\; \begin{cases} 0, & \text{if } \gamma \,\in\, \{0,1\}^m, \;\; m - \mathbf{1}^T\gamma \;=\; K \\ \infty, & \text{otherwise} \end{cases}
\tag{3}
$$

$$
\phi_2(z) = \begin{cases} 0, & \text{if} \quad P_l \,\leq\, P_l \,+\, z_l \,\leq\, 0 \\ & \text{and } 0 \;\leq\, P_g \,+\, z_g \,\leq\, P_g \\ \infty, & \text{otherwise} \end{cases}
\tag{4}
$$

and

$$
\phi_3(\theta) = \begin{cases} 0, & \text{if } -\dfrac{\pi}{2} \,\leq\, E^T\theta \,\leq\, \dfrac{\pi}{2} \\ \infty, & \text{otherwise.} \end{cases}
\tag{5}
$$

With these indicator functions, we can compactly express the minimization problem of the augmented Lagrangian (2) as

$$
\begin{array}{ll}
\underset{\theta,\,z,\,\gamma,\,\Gamma}{\text{minimize}} & \mathcal{L}_\rho(\theta,z,\gamma,\Gamma,\lambda) \;=\; \phi_1(\gamma) + \phi_2(z) + \phi_3(\theta) \\
& \quad +\, \mathbf{1}^T z_l \,+\, \dfrac{\rho}{2}\,\|c(\theta,z,\gamma) \,+\, \lambda/\rho\|_2^2,
\end{array}
$$

where we used the completion of squares and omitted the constant terms in λ.

We can now present the alternating method in Algorithm 1.

Algorithm 1 An ADMM algorithm for (2).

1: Start with an initial guess $(\gamma^0, z^0, \theta^0, \lambda^0)$ and set $k \leftarrow 0$.
2: **repeat**
3: $\quad \gamma^{k+1} \;:=\; \text{argmin}_\gamma\, \mathcal{L}_\rho(\gamma, z^k, \theta^k, \lambda^k)$
4: $\quad z^{k+1} \;:=\; \text{argmin}_z\, \mathcal{L}_\rho(\gamma^{k+1}, z, \theta^k, \lambda^k)$
5: $\quad \theta^{k+1} \;:=\; \text{argmin}_\theta\, \mathcal{L}_\rho(\gamma^{k+1}, z^{k+1}, \theta, \lambda^k)$
6: $\quad \lambda^{k+1} \;=\; \lambda^k + \rho\, c(\gamma^{k+1}, z^{k+1}, \theta^{k+1})$
7: **until** The stopping criterion (6) is satisfied.

We stop the ADMM algorithm when both the primal and the dual residuals are sufficiently small:

$$
\begin{aligned}
\|c(\theta^{k+1}, z^{k+1}, \gamma^{k+1})\| \;&\leq\; \epsilon_{\text{prim}}, \\
\|\theta^{k+1} - \theta^k\| + \|z^{k+1} - z^k\| + \|\gamma^{k+1} - \gamma^k\| \;&\leq\; \epsilon_{\text{dual}}.
\end{aligned}
\tag{6}
$$

Recall that the primal residual $\|c(\theta,z,\gamma)\|$ determines the solution accuracy for the power flow equations. We use an absolute and relative criterion [13],

$$
\begin{aligned}
\epsilon_{\text{prim}} &= \sqrt{n}\,\epsilon_{\text{abs}} + \epsilon_{\text{rel}} \max(\|r^k\|_2, \|z^k\|_2, \|P\|_2), \\
\epsilon_{\text{dual}} &= \sqrt{n}\,\epsilon_{\text{abs}} + \epsilon_{\text{rel}} \|\lambda^k\|_2,
\end{aligned}
$$

where

$$
r^k \;:=\; ED\,\text{diag}(\gamma^k)\sin(E^T\theta^k)
$$

and n is the number of buses.

Since the relaxed load-shedding problem (2) is still a nonconvex optimization problem with binary variables, the ADMM algorithm may not converge, for example, when ρ is not sufficiently large. In that case ADMM cycles through two or more feasible solutions with the same number of nonzero elements in γ. In numerical experiments in Section V, the positive scalar $\rho = 10^4$ works well. The ADMM algorithm typically converges in tens of iterates.

We next elaborate on the minimization steps for γ, z, and θ.

A. The γ-Minimization Step

Let $\delta = \mathbf{1} - \gamma$. Then the γ-minimization problem can be expressed as

$$\begin{array}{ll} \text{minimize} & q(\delta) \,=\, \frac{\rho}{2} \left\| c(\theta^k, z^k, \mathbf{1} - \delta) + \lambda^k/\rho \right\|_2^2 \\ \text{subject to} & \delta \,\in\, \{0,1\}^m, \quad \mathbf{1}^T\delta \,=\, K. \end{array}$$

With some algebra, we can rewrite the quadratic objective function as

$$q(\delta) \,=\, \frac{\rho}{2} \left\| M^k\delta \,-\, b^k \right\|_2^2,$$

where

$$M^k \,:=\, ED\,\mathrm{diag}(\sin(E^T\theta^k))$$

and

$$b^k \,:=\, M^k\mathbf{1} \,+\, \lambda^k/\rho \,-\, (P \,+\, z^k).$$

Therefore, the γ-minimization step can be interpreted as finding K columns of matrix M^k such that their summation is closest to b^k. This combinatorial problem has been studied extensively in the context of signal recovery; see [17], [18].

We use a simple greedy heuristic that works well in practice [17]: Find the column of M^k that is closest to b^k, subtract that column from b^k, and repeat until all K columns have been chosen. It was shown in [18] that the global solution can be found via the greedy algorithm under certain orthogonality conditions on the coefficient matrix M^k.

B. The z-Minimization Step

With some algebra, one can show that the z-minimization problem can be expressed as

$$\begin{array}{ll} \text{minimize} & -\mathbf{1}^T z_l + \frac{\rho}{2} \left\| V_l^k - z_l \right\|_2^2 + \frac{\rho}{2} \left\| V_g^k - z_g \right\|_2^2 \\ \text{subject to} & P_l \,\le\, P_l + z_l \,\le\, 0,\; 0 \,\le\, P_g + z_g \,\le\, P_g, \end{array}$$

where

$$V^k \,:=\, ED\sin(E^T\gamma^{k+1}) \,+\, \lambda^k/\rho \,-\, z$$

and $V^k = [\,(V_l^k)^T\;(V_g^k)^T\,]^T$ is partitioned with respect to the load and the generation variables. This convex quadratic program with simple box constraints has the following analytical solution:

$$(z_g)_i \,=\, \begin{cases} (V_g^k)_i, & \text{if } -(P_g)_i \,\le\, (V_g^k)_i \,\le\, 0 \\ 0, & \text{if } (V_g^k)_i \,<\, -(P_g)_i \\ -(P_g)_i, & \text{if } (V_g^k)_i \,>\, 0 \end{cases}$$

and

$$(z_l)_i \,=\, \begin{cases} (V_l^k)_i + \frac{1}{\rho}, & \text{if } 0 \,\le\, (V_l^k)_i + \frac{1}{\rho} \,\le\, -(P_l)_i \\ 0, & \text{if } (V_l^k)_i + \frac{1}{\rho} \,<\, 0 \\ -(P_l)_i, & \text{if } (V_l^k)_i \,>\, -(P_l)_i, \end{cases}$$

where we use $(\cdot)_i$ to denote the ith element of a vector.

C. The θ-Minimization Step

The θ-minimization problem can be expressed as follows:

$$\begin{array}{ll} \text{minimize} & \frac{\rho}{2} \left\| c(\theta, z^{k+1}, \gamma^{k+1}) + \lambda^k/\rho \right\|_2^2 \\ \text{subject to} & -\frac{\pi}{2} \,\le\, E^T\theta \,\le\, \frac{\pi}{2}. \end{array}$$

Let us denote

$$C^k \,:=\, ED\,\mathrm{diag}(\gamma^{k+1}),$$
$$d^k \,:=\, P \,+\, z^{k+1} \,-\, \lambda^k/\rho,$$

and

$$y \,:=\, E^T\theta.$$

Then we can rewrite the θ-minimization problem as

$$\begin{array}{ll} \text{minimize} & \frac{\rho}{2} \left\| C^k\sin(y) \,-\, d^k \right\|_2^2 \\ \text{subject to} & -\frac{\pi}{2} \,\le\, y \,\le\, \frac{\pi}{2}. \end{array}$$

With a change of variable $x := \sin(y)$, we get

$$\begin{array}{ll} \text{minimize} & \frac{\rho}{2} \left\| C^k x \,-\, d^k \right\|_2^2 \\ \text{subject to} & -1 \,\le\, x \,\le\, 1. \end{array}$$

This is a bound-constrained least-squares problem that can be solved efficiently, for example, by the two-metric projection method [19] or the trust-region methods implemented in quadratic programming solvers in Matlab. Having found the solution x, we take

$$y \,=\, \arcsin(x),$$

which yields a unique solution in the interval $[-\pi/2, \pi/2]$. We then compute

$$\theta \,=\, (E^T)^\dagger y,$$

where $(E^T)^\dagger$ is the pseudoinverse of E^T.

V. Numerical Results

We use the IEEE 118-bus system as a test case for the minimum load-shedding problem (1). Figure 1 shows a diagram of the IEEE 118-bus test case. This power system has 54 generator buses, 64 load buses, and 186 transmission lines. We obtain the generation P_g and load P_l profiles by solving the steady-state power flow equations via MATPOWER [20].

Our numerical experiments are performed on a workstation with 32 GB memory and two Intel E5430 Xeon 4-core 2.66 GHz CPUs running Matlab R2013a in Ubuntu 12.04. We set the maximum number of ADMM steps to be MaxIter $= 10^3$, the relative stopping criterion to be $\epsilon_{\mathrm{rel}} = 10^{-6}$, the absolute stopping criterion to be $\epsilon_{\mathrm{abs}} = 10^{-4}$, and the positive scalar to be $\rho = 10^4$. We observe that the ADMM algorithm typically converges in tens of iterates.

TABLE I: load-shedding strategy for the IEEE 118-bus test case. The lines to be removed are highlighted in Fig. 2 and Fig. 3.

K	Load Shed (MW)	Percentage	Lines Removed
1	0	0	73
2	2.3735	0.054%	73, 146
3	3.6870	0.084%	73, 88, 146
4	6.1867	0.141%	59, 73, 88, 146
5	9.1851	0.230%	59, 73, 88, 146, 185

We start by solving the minimum load-shedding problem (1) with $K = 1$, that is, removing only one line. The IEEE 118-bus test case turns out to have enough redundancy in the system that removing line $l = 73$ (highlighted in Fig. 2) does not result in shedding any load. In other words, the power network has enough capacity to route the same amount of power flow without using line $l = 73$.

Since the load shedding is zero for $K = 1$, we have proved that the global solution for the MINLP (1) has been found by the ADMM algorithm.

We next set $K = 2$, that is, removing two transmission lines. In this case, the amount of load to be shed is 2.3735 MW, which is 0.054% of the total load 4374.9 MW. The lines to be removed are $l = 73$ and $l = 146$, highlighted in Fig. 2 and Fig. 3, respectively.

As we increase K, the amount of load-shedding increases. The computational results for $K = 1, 2, 3, 4, 5$ are summarized in Table I. Note that the set of lines to be removed for K contains the set of lines for $K - 1$. While the selected lines may not be the optimal solutions of (1), they are feasible solutions, therefore providing an upper bound on the optimal value of (1).

Figure 2 and Figure 3 show the northeast and the southeast corner of the system, respectively. We highlight the set of lines to be taken out of service. Lines 59, 73, and 88 are highlighted, respectively, in blue, yellow, and green color in Fig. 2; Lines 146 and 185 are highlighted, respectively, in red and purple color in Fig. 3. The reason that removing line 73 does not result in load-shedding is that it connects two load buses 52 and 53, both of which have direct connections to generator buses; see Fig. 2.

Table II shows the type of buses connected by the out-of-service lines. Note that all buses in Table II are load buses except bus 59, which is a generator bus. This result suggests that taking lines between load buses out of service leads to less load-shedding.

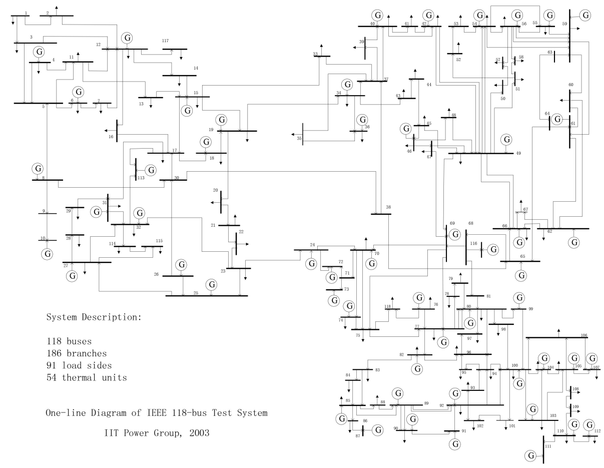

Fig. 1: Diagram of the IEEE 118-bus test case.

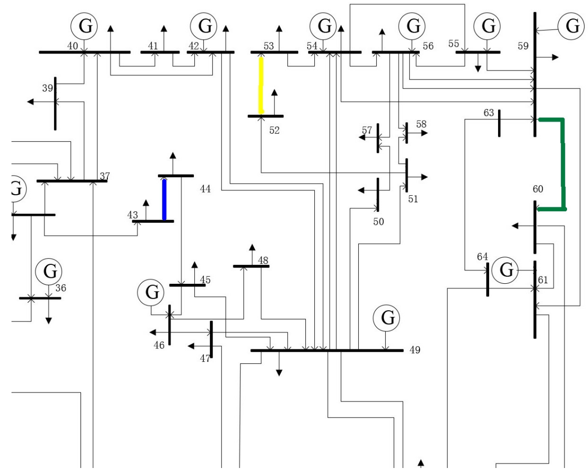

Fig. 2: Northeast corner of the IEEE 118-bus test case. Lines 59, 73, and 88 to be taken out of service are highlighted, respectively, in blue, yellow, and green color.

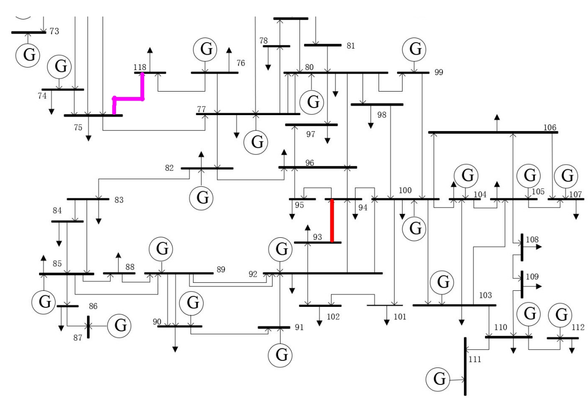

Fig. 3: Southeast corner of the IEEE 118-bus test case. Lines 146 and 185 to be taken out of service are highlighted in red and purple color, respectively.

TABLE II: Set of lines and the bus types. Lines 59, 73, and 88 are highlighted in Fig. 2. Lines 146 and 185 are highlighted in Fig. 3.

Line	Bus	Type	Bus	Type	Color	Figure
59	43	load	44	load	blue	2
73	52	load	53	load	yellow	2
88	59	gen	60	load	green	2
146	93	load	94	load	red	3
185	75	load	118	load	purple	3

VI. Conclusions

In this paper, we formulate the minimum load-shedding problem for electric power systems. Our objective is to choose a prespecified number of lines to be taken offline to minimize the amount of load shed. The AC power flow equations and the binary decision variables lead to a mixed-integer nonlinear program. We show that this challenging problem has a separable structure that can be exploited by an ADMM algorithm. Numerical experiments on the IEEE 118-bus test case demonstrate the effectiveness of the developed approach. In particular, when a single line is to be taken out, the ADMM algorithm finds the global solution. Numerical results suggest that selecting lines that connect load buses results in less load shedding.

References

[1] G. Andersson, P. Donalek, R. Farmer, N. Hatziargyriou, I. Kamwa, P. Kundur, N. Martins, J. Paserba, P. Pourbeik, J. Sanchez-Gasca *et al.*, "Causes of the 2003 major grid blackouts in North America and Europe, and recommended means to improve system dynamic performance," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 1922–1928, 2005.

[2] C. D. Brummitt, R. M. D'Souza, and E. Leicht, "Suppressing cascades of load in interdependent networks," *Proceedings of the National Academy of Sciences*, vol. 109, no. 12, pp. E680–E689, 2012.

[3] F. D. Galiana, "Bound estimates of the severity of line outages in power system contingency analysis and ranking," *IEEE Transactions on Power Apparatus and Systems*, no. 9, pp. 2612–2624, 1984.

[4] V. Donde, V. López, B. Lesieutre, A. Pinar, C. Yang, and J. Meza, "Identification of severe multiple contingencies in electric power networks," in *Proceedings of the 37th Annual North American Power Symposium*, 2005, pp. 59–66.

[5] ——, "Severe multiple contingency screening in electric power systems," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 406–417, 2008.

[6] A. Pinar, J. Meza, V. Donde, and B. Lesieutre, "Optimization strategies for the vulnerability analysis of the electric power grid," *SIAM Journal on Optimization*, vol. 20, no. 4, pp. 1786–1810, 2010.

[7] K. W. Hedman, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Optimal transmission switching with contingency analysis," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1577–1586, 2009.

[8] M. Khanabadi, H. Ghasemi, and M. Doostizadeh, "Optimal transmission switching considering voltage security and N-1 contingency analysis," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 542–550, 2013.

[9] F. Li, W. Qiao, H. Sun, H. Wan, J. Wang, Y. Xia, Z. Xu, and P. Zhang, "Smart transmission grid: Vision and framework," *IEEE Transactions on Smart Grid*, vol. 1, no. 2, pp. 168–177, 2010.

[10] R. Fletcher and S. Leyffer, "Solving mixed-integer nonlinear programs by outer approximation," *Mathematical Programming*, vol. 66, no. 1-3, pp. 327–349, 1994.

[11] S. Leyffer, "Integrating SQP and branch-and-bound for mixed-integer nonlinear programming," *Computational Optimization and Applications*, vol. 18, no. 3, pp. 295–309, 2001.

[12] K. Abhishek, S. Leyffer, and J. Linderoth, "FILMINT: An outer approximation-based solver for convex mixed-integer nonlinear programs," *INFORMS Journal on Computing*, vol. 22, no. 4, pp. 555–567, 2010.

[13] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2011.

[14] F. Lin, M. Fardad, and M. R. Jovanović, "Design of optimal sparse feedback gains via the alternating direction method of multipliers," *IEEE Transactions on Automatic Control*, vol. 58, no. 9, pp. 2426–2431, 2013.

[15] ——, "Algorithms for leader selection in stochastically forced consensus networks," *IEEE Transactions on Automatic Control*, vol. 59, no. 7, pp. 1789–1802, 2014.

[16] F. Lin, M. R. Jovanović, and T. T. Georgiou, "An ADMM algorithm for matrix completion of partially known state covariances," in *Proceedings of the 52nd Conference on Decision and Control*, 2013, pp. 1684–1689.

[17] Y. C. Pati, R. Rezaiifar, and P. Krishnaprasad, "Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition," in *The Twenty-Seventh Asilomar Conference on Signals, Systems and Computers*, 1993, pp. 40–44.

[18] J. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Transactions on Information Theory*, vol. 53, no. 12, pp. 4655–4666, 2007.

[19] E. M. Gafni and D. P. Bertsekas, "Two-metric projection methods for constrained optimization," *SIAM Journal on Control and Optimization*, vol. 22, no. 6, pp. 936–964, 1984.

[20] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "Matpower: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2011.